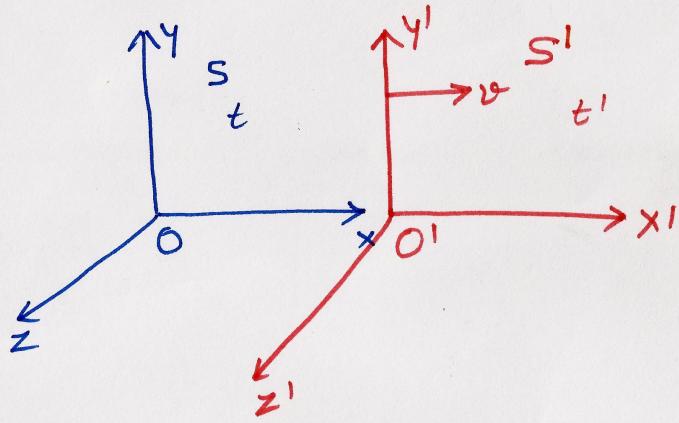


## Frame of Reference



In  $S'$  frame, observer  $O'$   
( $x', y', z', t'$ )

In  $S$  frame, observer  $O$   
( $x, y, z, t$ )

S-Stationary frame

Type of frame of Reference

- 1) Inertial - unaccelerated
- 2) Non-Inertial - accelerated

1) Newton's Laws hold good in inertial frame of Reference

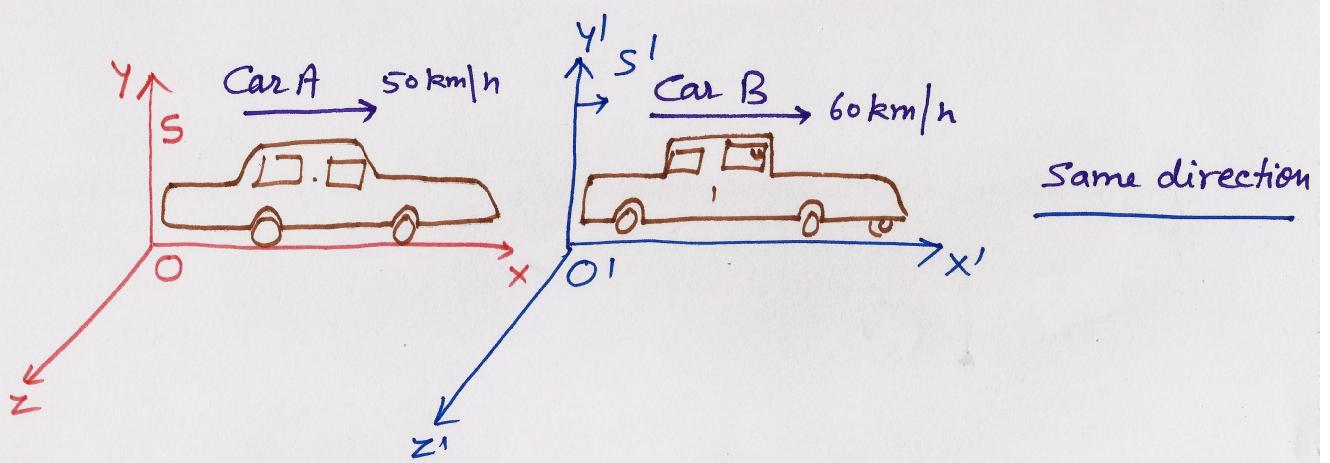
2) Newton's Laws do not hold good in non-inertial frame of reference.

Relative motion?

i)  $\xrightarrow{A} \xrightarrow{B}$

ii)  $B \xleftarrow{A}$

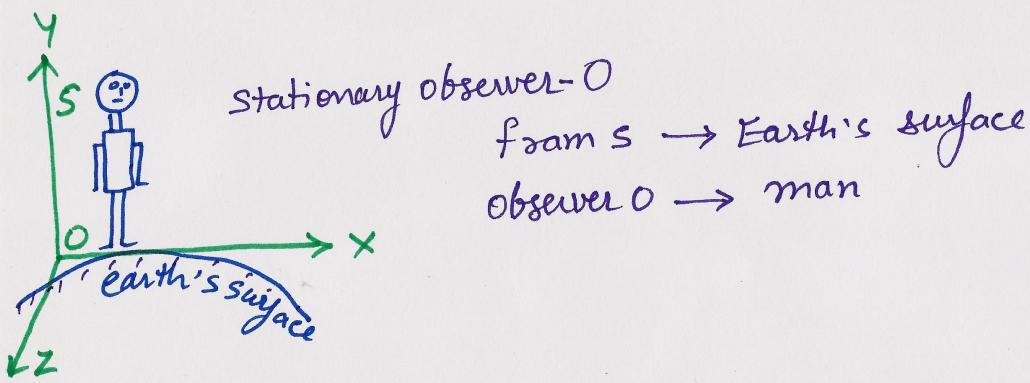
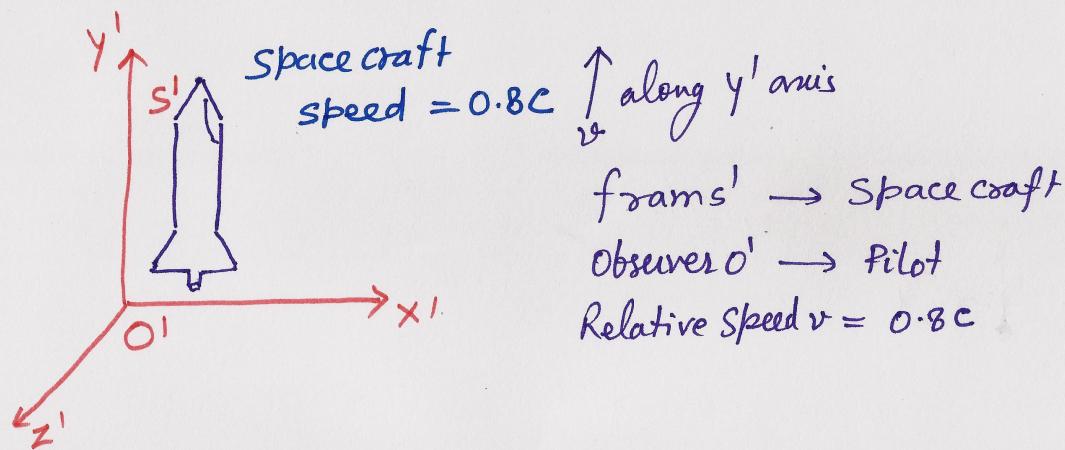
## Examples of frame of References



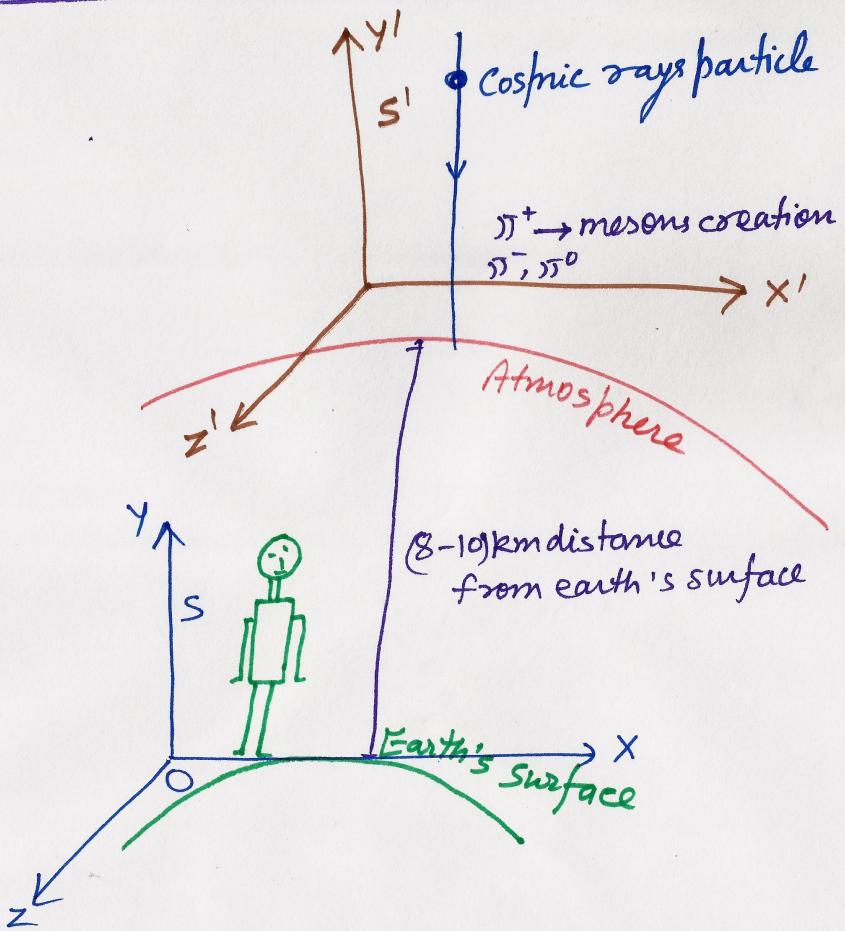
Car A, speed 50 km/h  
observer O; Driver of Car A  
frame S; Car A  
direction; +ve x axis

car B, Speed 60 km/h  
observer  $O'$ ; Driver of car B  
frame  $S'$ ; Car B  
direction; +ve  $x'$  axis.

## Example frame of Reference:



## Example frame of Reference



frame  $S' \rightarrow \pi^+$  meson particles

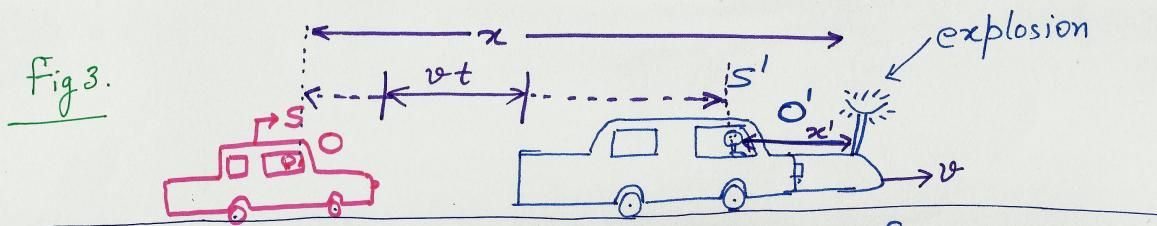
Speed  $v = 2.992 \times 10^8 \text{ m/s}$

frame S - Earth's Surface  
observer O - man

D

### Galilean Transformation :-

Fig 3.



PINK CAR =  $S$  frame  
Pink Car Driver =  $O$  observer

Blue car =  $S'$  frame  
Driver =  $O'$  observer  
Explosion =  $P$  event  
Velocity =  $v$  relative to  $S$ ; Constant

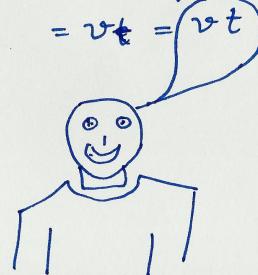
\* Observation of any event for observers  $O$  and  $O'$  ?

- \* According to  $\underline{O}$  in frame  $S$ ;  $(x, y, z, t)$
- \* According to  $\underline{O}'$  in frame  $S'$ ;  $(x', y', z', t')$

\* TRANSFORMATION EQUATIONS

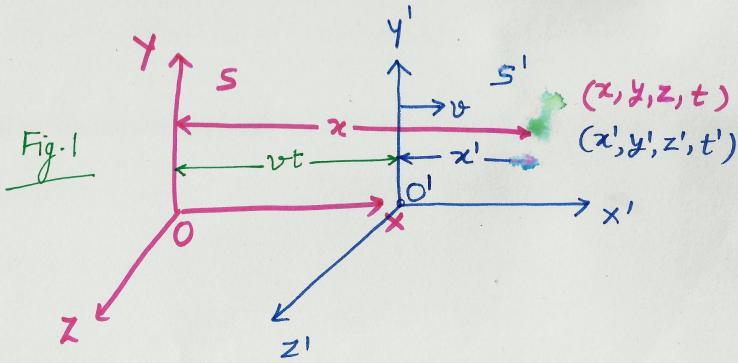
$$\begin{array}{l|l} x = x' + vt & x' = x - vt \\ y = y' & y' = y \\ z = z' & z' = z \\ t = t' & t' = t \end{array}$$

\* At any time  $t$ , The distance between pink & Blue car = velocity  $\times$  time  
 $= vt = vt$



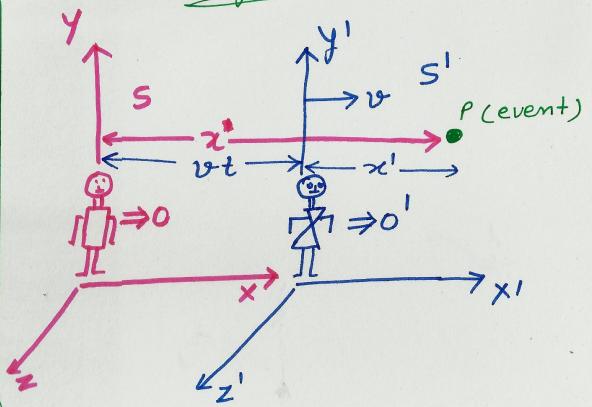
②

## Galilean Transformation :



- \* Frame of References ;  $S$  and  $S'$
- \* Observers;  $O$  and  $O'$
- \* Event  $P$
- \* Observation of  $O \& O'$  observers  
 $(x, y, z, t) \& (x', y', z', t')$
- \* Relative motion  $\rightarrow$  along +ve  $x$  axis  
 $\rightarrow$  along  $x'$

Fig. 2.

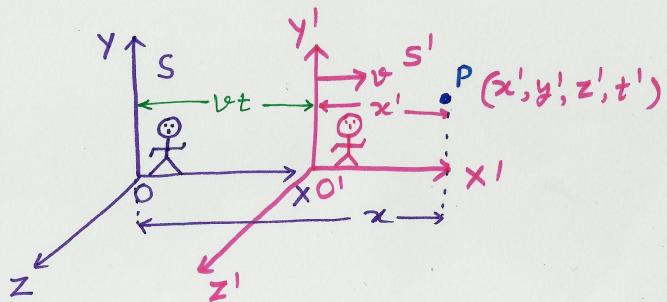


- \*  $P$  event occur in  $S'$  frame
  - \*  $S'$  frame moving frame with velocity  $v$  relative to  $S$ .
  - \* According to 'O' observer; the coordinates of event  $P$ . -  $(x, y, z, t)$
  - \* According to  $O'$  observer; the coordinates of event  $P$  -  $(x', y', z', t')$
- Observation → How you see that event? Appearance of shape, size and time
- At what time that event happens? and Time

③

### Galilean Transformation

[Case I ; motion along +ve xx'-direction]  
Transform the coordinates of a particle  
from one inertial system to another.



O = Saniya  
O' = Praney

S frame ; observer O | S' frame ; Observer O'

$$x = x' + vt$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

y

### Galilean Transformation ; motion along +ve x-direction (+xx')

In S-frame :-

$$x = x' + vt \Rightarrow \frac{dx}{dt} = \frac{dx'}{dt} + v \frac{dt}{dt}$$

$$y = y'$$

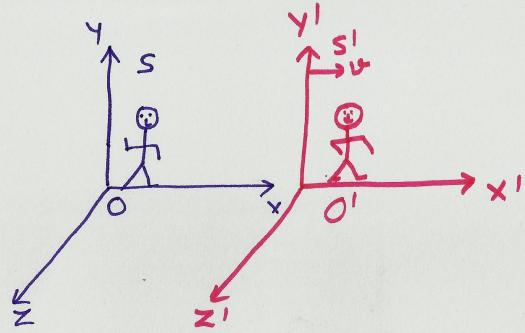
$$z = z'$$

$$t = t'$$

$$u_x = \frac{dx}{dt}; \quad u'_x = \frac{dx'}{dt'}$$

$$t = t'$$

$$\frac{d}{dt} = \frac{d}{dt'}$$



So,

$u_x = u'_x + v$
$u_y = u'_y$
$u_z = u'_z$

①



In S' - frame

$$x' = x - vt \Rightarrow \frac{dx'}{dt'} = \frac{dx}{dt} - v \frac{dt}{dt'}$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

$$u'_x = \frac{dx'}{dt'}; \quad u_x = \frac{dx}{dt}$$

So

$u'_x = u_x - v$
$u'_y = u_y$
$u'_z = u_z$

②

5

## Galilean Transformation

### Galilean acceleration Transformation

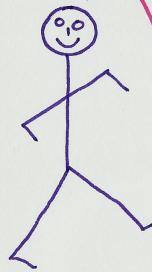
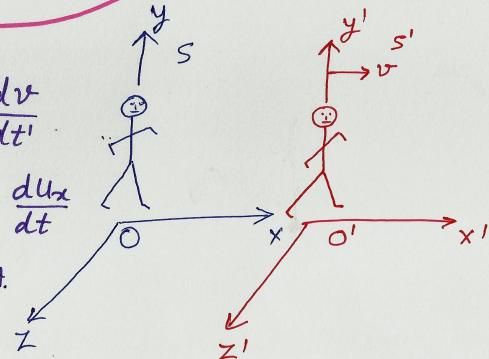
$v$  = velocity constant



$$\begin{aligned} u_x' &= u_x - v \Rightarrow \frac{du_x'}{dt'} = \frac{du_x}{dt} - \frac{dv}{dt'} \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}$$

$$a_x' = \frac{du_x'}{dt'} ; a_x = \frac{du_x}{dt}$$

$$\frac{dv}{dt} = 0 ; v = \text{const.}$$



Same Acceleration  
in  $S$  and  $S'$  frame

$$\begin{array}{|c|} \hline a_x' = a_x \\ \hline a_y' = a_y \\ \hline a_z' = a_z \\ \hline \end{array}$$



Both  $S$  and  $S'$  frames  
are inertial

6

## Galilean Transformation

$S$ -frame observer  $O$

$$x = x' + vt$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

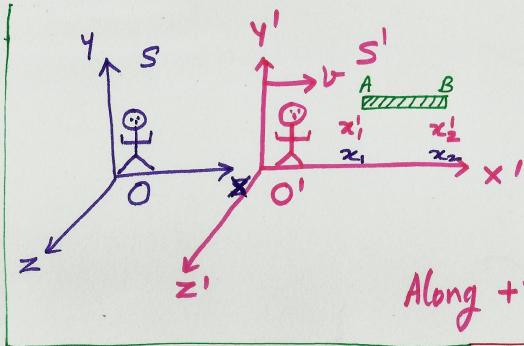
$S'$ -frame;  $O'$  observer

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$



Along +ve  $x$   $x'$

In  $S'$ -frame

A Rod AB of length  $l_0$

$O'$ : observer's observation

for point A;  $x'_1, y'_1, z'_1, t'_1$

B;  $x'_2, y'_2, z'_2, t'_2$

proper length of the rod

$$l_0 = x'_2 - x'_1$$

In  $S$ -frame

$O$ : observer's observation  
for the end A of rod;  $x_1, y_1, z_1, t_1$   
for the end B of the rod;  $x_2, y_2, z_2, t_2$

$$\text{Length of the rod} = x_2 - x_1$$

Acc. to observer  $O$ , the length of the

$$\text{rod} = x_2 - x_1$$

$$= (x'_2 - vt'_1) - (x'_1 - vt'_1)$$

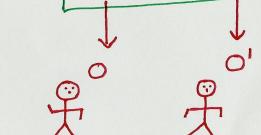
$$= (x'_2 - x'_1) - v(t'_2 - t'_1)$$

$$x_2 - x_1 = (x'_2 - x'_1)$$

\* No change in shape

\* No change in time.

$$x_2 - x_1 = x'_2 - x'_1$$



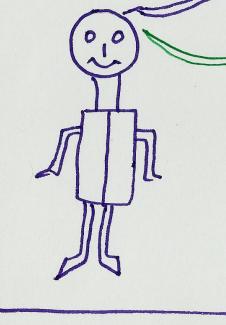
Same Length

① ⑦

## Galilean Transformation

[Case II - motion in any direction]

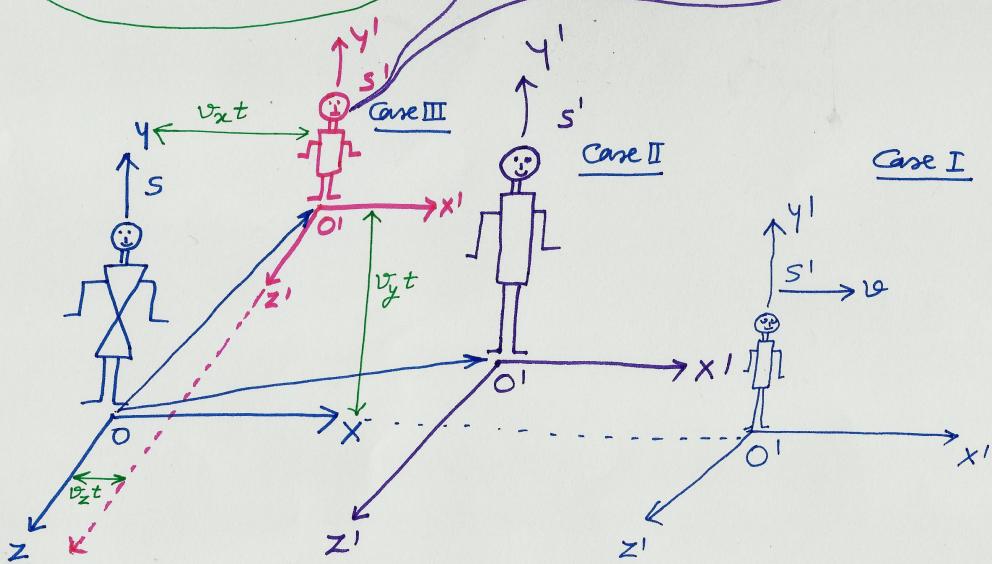
\* It is not necessary that  $s'$  moves, always along +ve  $x$ -axis



What changes comes  
and where?

velocity Components  
 $v_x, v_y, v_z ?$

O; Saniya  
O'; Praney



⑧

## Galilean Transformation

$S'$ -frame in any direction  
(Case II)

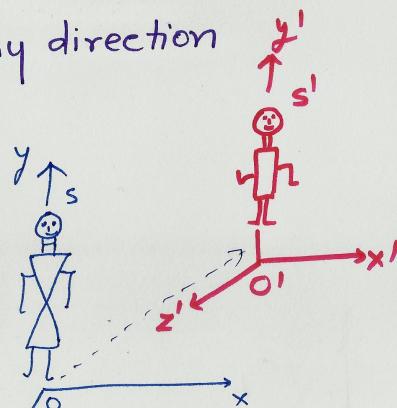
$O = \text{Saniya}$   
 $O' = \text{Praney}$

In  $S$ -frame  
Saniya's observation

$$\begin{aligned}x &= x' + v_x t \\y &= y' + v_y t \\z &= z' + v_z t \\t &= t'\end{aligned}$$

In  $S'$ -frame  
Praney's observation

$$\begin{aligned}x' &= x - v_x t \\y' &= y - v_y t \\z' &= z - v_z t \\t' &= t\end{aligned}$$



Galilean velocity transformation  $\rightarrow$

in  $S'$ -frame

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v_x \frac{dt}{dt'} \quad [t' = t \Rightarrow \frac{dt}{dt'} = \frac{dt}{dt}]$$

$$u'_x = \frac{dx'}{dt'} ; u_x = \frac{dx}{dt} \quad \left| \frac{dt}{dt'} = \frac{dt}{dt} = 1 \right.$$

$$\therefore u'_x = u_x - v_x \quad \rightarrow ①$$

What Praney says?

$$\begin{aligned}y' &= y - v_y t \\ \frac{dy'}{dt'} &= \frac{dy}{dt} - v_y \frac{dt}{dt'} \quad \left| \begin{array}{l} t' = t \\ \frac{dt}{dt'} = \frac{dt}{dt} \end{array} \right. \\ u'_y &= u_y - v_y \quad \rightarrow ② \\ u'_z &= u_z - v_z \end{aligned}$$

$$\left| \begin{array}{l} \frac{dy}{dt} = \frac{dy}{dt'} \\ u_y = \frac{dy}{dt'} \end{array} \right. \quad \left| \begin{array}{l} u'_y = \frac{dy}{dt'} \\ u_y = \frac{dy}{dt} \end{array} \right.$$

⑨

## Galilean Transformation

### Galilean acceleration Transformation



$v_x; v_y; v_z$  constants.

$$u'_x = u_x - v_x$$

$$\frac{du'_x}{dt'} = \frac{du_x}{dt} - \frac{dv_x}{dt}$$

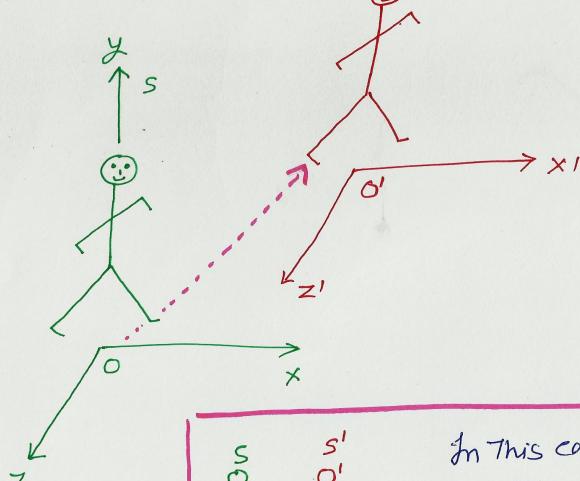
$$a'_x = \frac{du'_x}{dt'}; a_x = \frac{du_x}{dt}$$

$$t = t' \Rightarrow \frac{d}{dt} = \frac{d}{dt'}$$

$$a'_x = a_x$$

$$a'_y = a_y; a'_z = a_z$$

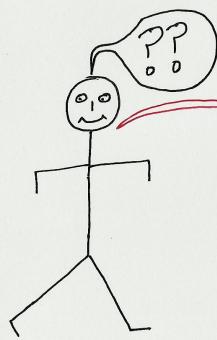
$s'$ -frame in any direction



In this case also,  
both the frame  
 $s$  and  $s'$  are  
inertial

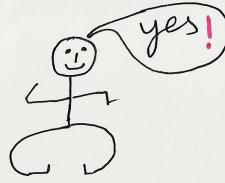
10

## Failure of Galilean Transformation



Where is the problem?

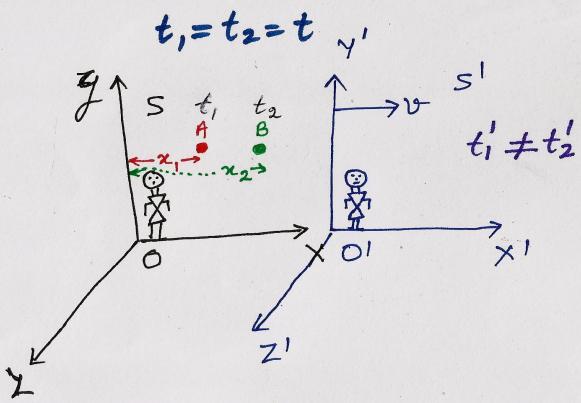
Are all the laws of physics invariant  
in all inertial frame of References?



Are the laws of electricity  
and Magnetism the same  
in inertial frame of references?



## Relativity of Simultaneity :-



- Time is not absolute
- Length is not absolute.

$$\Delta t' = t'_2 - t'_1 = ? \quad [\text{Find it?}]$$

Two events A & B occurring simultaneously

\* In frame S,

Coordinates for the event A  
Acc. to observer O;  $(x_1, y_1, z_1, t_1)$

Coordinates for the event B  
Acc. to observer O;  $(x_2, y_2, z_2, t_2)$

$$\text{But } t_1 = t_2 = t \\ \therefore \Delta t = t_2 - t_1 = 0$$

\* In frame S':

Coordinates for the event A  
Acc. to observer O';  $(x'_1, y'_1, z'_1, t'_1)$

Coordinates for the event B  
According to observer O';  
 $(x'_2, y'_2, z'_2, t'_2)$

$$\text{Here } t'_1 \neq t'_2$$

## Relativity of Simultaneity... (2)

$$\Delta t' = t_2' - t_1'$$

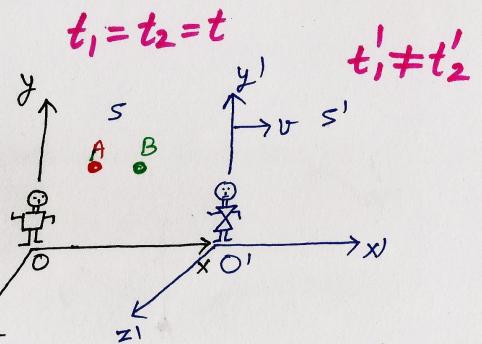
time interval between two events  
Acc. to  $O'$  observer.

From Lorentz's Transformation

$$t_1' = \frac{t_1 - \frac{x_1 v}{c^2}}{\sqrt{1 - v^2/c^2}} ; t_2' = \frac{t_2 - \frac{x_2 v}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t' = t_2' - t_1' = \frac{t_2 - \frac{x_2 v}{c^2}}{\sqrt{1 - v^2/c^2}} - \frac{t_1 - \frac{x_1 v}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t' = \frac{(t_2 - t_1) + (x_1 - x_2) \frac{v}{c^2}}{\sqrt{1 - v^2/c^2}} = \frac{(x_1 - x_2) \frac{v}{c^2}}{\sqrt{1 - v^2/c^2}}$$



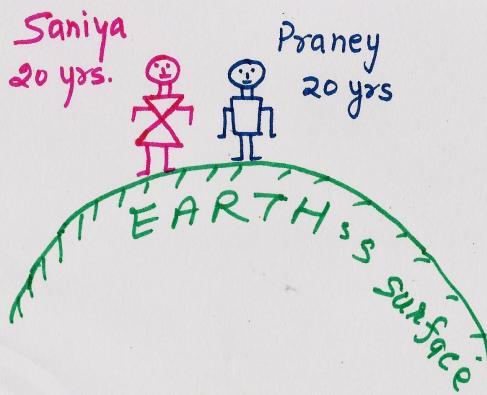
$$\boxed{\Delta t' = \frac{(x_1 - x_2) \frac{v}{c^2}}{\sqrt{1 - v^2/c^2}}}$$

$$\boxed{\Delta t' \propto (x_1 - x_2)}$$

## Twin Paradox :-

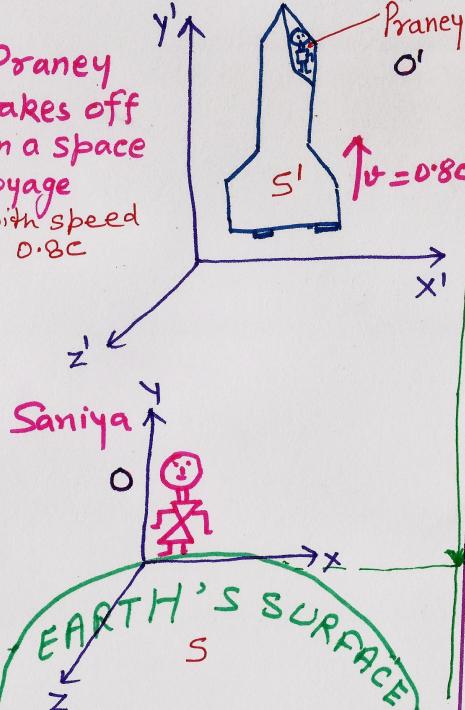
### Situation - I<sup>st</sup>

- \* A pair of twins; Saniya and Praney
- Age = 20 yrs.



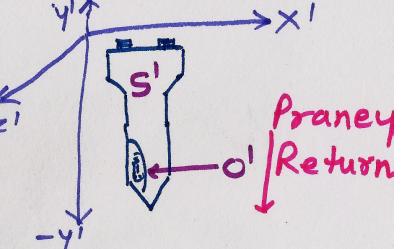
### Situation - II<sup>nd</sup>

Praney takes off on a space voyage with speed  $0.8c$

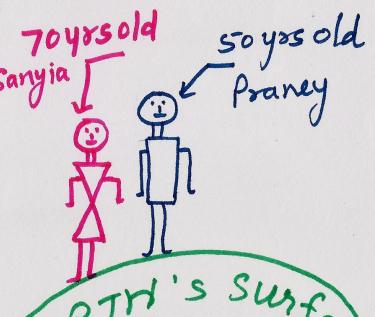


Star  
20 light years away from earth

### Situation III<sup>rd</sup>



### Situation IV<sup>th</sup>



## Twin Paradox

Praney is 20 yrs old when he takes off on a space voyage at a speed of  $0.80c$  to a star 20 light years away.

To Saniya, who stays behind, the pace of Praney's life is slower than her by a factor

$$\sqrt{1-v^2/c^2} = 0.60 \Rightarrow 60\%$$

(A moving clock ticks more slowly than a clock at rest)

For Saniya, everything will be 60% of her action.

Praney's heart beats only 3 times for every 5 beats of her heart.

Praney takes only 3 breaths for every 5 of hers.

Saniya says that Praney returns after 50 yrs but Praney takes only 30 yrs for the round trip.

NOTE! What is important is here?

→ Praney's non-inertial frame!

\* Praney thinks that he covers only the distance  $L$ ;

$$L = L_0 \sqrt{1-v^2/c^2} = 20 \text{ yrs} \sqrt{1-\frac{0.80^2}{c^2}}$$

$$L = 12 \text{ light years}$$

his voyage to the star takes time  $= \frac{L}{v} = 15 \text{ yrs.}$

and same for return voyage.

Total = 30 yrs.

Saniya says he spent 50 yrs. according to his calendar.

The Conclusion is ~~that~~  
Praney covers the distance in his frame of Reference and that is shortened to 12 yrs.