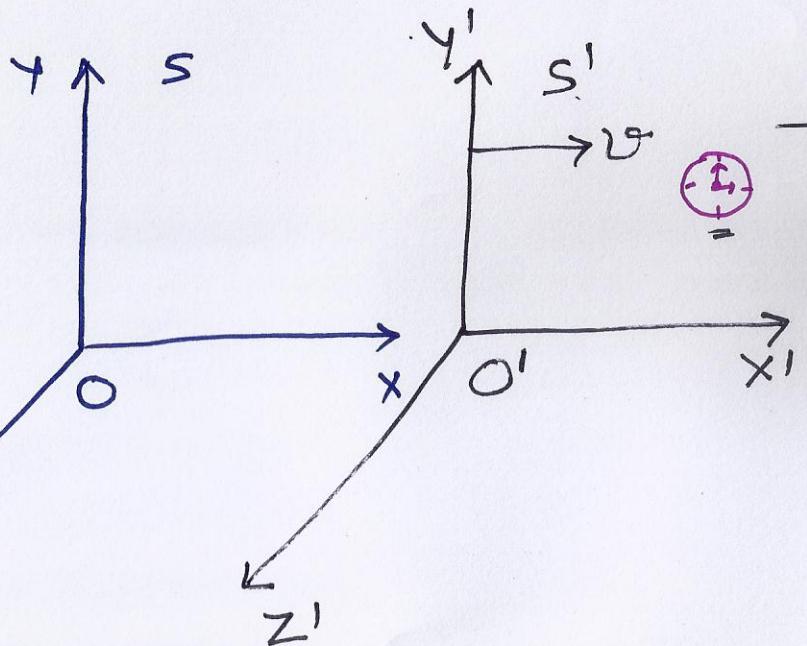


①

Time Dilation



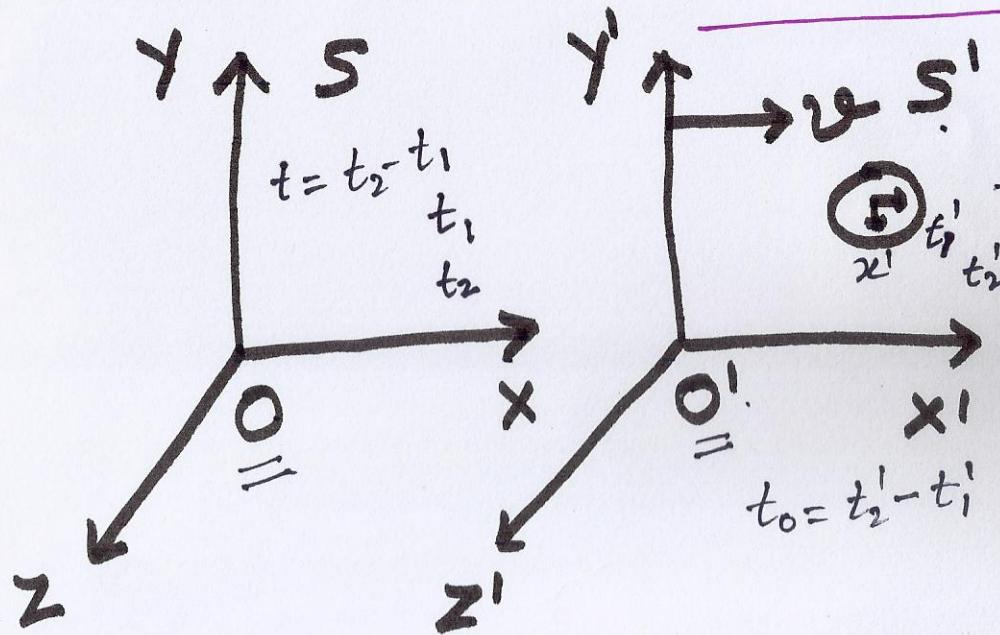
* A clock in a moving +ve x frame will be seen to be running slow or 'dilated' according to the Lorentz Transformation.

* A moving clock ticks more slowly than a clock at rest

* The time will always be shortest as measured in its rest frame.

* The time measured in the frame in which the clock is at rest is called the "proper time"

②



The time measurements made in the moving frame are made at the same location,

$$t = t_2 - t_1 = t'_2 + \frac{x'_2 v}{c^2} - t'_1 - \frac{x'_1 v}{c^2}$$

$$\frac{t'_2 - t'_1}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t'_2 - t'_1}{\sqrt{1 - v^2/c^2}} \Rightarrow \boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$$

Time dilation

Let a clock be fixed at the point x' in the moving frame S' . An observer in S' finds that the clock produces two ticks at time t'_1 and t'_2 . The time interval between these ticks as measured in the frame S' is given by

$$t_0 = t'_2 - t'_1 \quad \text{--- (1)}$$

Then in rest frame

$$t = t_2 - t_1 \quad \text{--- (2)}$$

by L.T. (Lorentz transformation)

$$t = \frac{t'_1 + \frac{x'_1 v}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Time Dilation

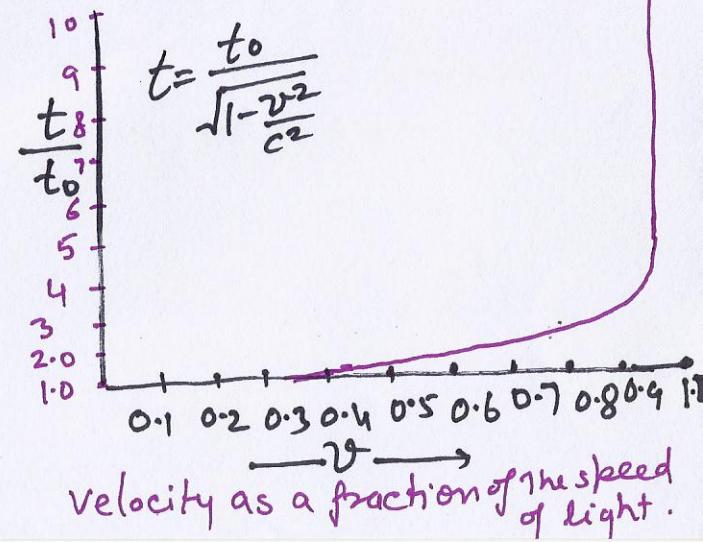
$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

The mean lifetime of a muon in its own reference frame, called the proper lifetime, is $t_0 = 2.2 \mu s$. In a frame moving at velocity v with respect to that proper frame, the lifetime t is

proper time $= 2.2 \times 10^{-6} \text{ sec.}$

$\mu \rightarrow v=0 \quad t_0 = 2.2 \mu s$

Sr. No.	velocity	time dilation t	$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$
1.	$0.4c$	$1.0910 t_0$	$= 2.4002 \times 10^{-6} \text{ sec}$
2.	$0.6c$	$1.25 t_0$	$= 2.75 \times 10^{-6} \text{ sec.}$
3.	$0.8c$	$1.666 t_0$	$= 3.6652 \times 10^{-6} \text{ sec.}$
4.	$0.9c$	$2.294 t_0$	$= 5.0468 \times 10^{-6} \text{ sec.}$
5.	$0.998c$	$15.819 t_0$	$= 34.8018 \times 10^{-6} \text{ sec.}$
6.	c	∞	$= \infty \text{ secs.}$



IQ:- What is the mean life time of π^+ meson moving with speed of $\frac{0.73c}{v}$, when the proper life is 2.5×10^{-8} sec?

$$\text{Given} \Rightarrow v = 0.73c \quad c = \text{speed of light} \\ = 3 \times 10^8 \text{ m/s.}$$

$$t_0 = 2.5 \times 10^{-8} \text{ sec.}$$

$$\frac{v^2}{c^2} = \left(\frac{v}{c}\right)^2$$

$$(0.73)^2$$

$$= \frac{0.5329}{\sqrt{1 - 0.5329}}$$

$$= \sqrt{0.4671}$$

$$= \underline{\underline{0.6834}}$$

by using the formula $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.5 \times 10^{-8} \text{ sec}}{\sqrt{1 - \left(\frac{0.73c}{c}\right)^2}}$

$$= \frac{2.5 \times 10^{-8} \text{ sec}}{0.6834}$$

$$= 3.658 \times 10^{-8} \text{ sec.}$$

$$t \approx 3.7 \times 10^{-8} \text{ sec}$$

Q: What is the velocity of π mesons whose proper mean life is 2.5×10^{-8} sec and observed mean life is 2.5×10^{-7} sec?

Given $t_0 = 2.5 \times 10^{-8}$ s. ; $t = 2.5 \times 10^{-7}$ sec. $v = ?$

by using formula $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow t \sqrt{1 - \frac{v^2}{c^2}} = \frac{t_0}{t}$

$$1 - \frac{v^2}{c^2} = \left(\frac{t_0}{t}\right)^2 \Rightarrow \frac{v^2}{c^2} = 1 - \left(\frac{t_0}{t}\right)^2 \Rightarrow v = c \sqrt{1 - \left(\frac{t_0}{t}\right)^2}$$

3×10^8 m/s.

$$v = c \sqrt{1 - \left(\frac{2.5 \times 10^{-8}}{2.5 \times 10^{-7}}\right)^2} = c \sqrt{1 - 0.01} = 0.995 c$$

Ans.

3Q: A wrist watch keeping correct time on earth, is worn by the pilot of a spaceship. How much will it appear to lose per day with respect to an observer on earth when spaceship leaves the earth with a constant velocity of $10^7 \frac{m}{sec}$.

Given; 1 day = 24 hours ✓

$v = 10^7 \text{ m/s.} \quad t_0 = ?$ ✓

$$\tilde{t} = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 24 = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad t = 24 \text{ hours.}$$

$$24 = \frac{t_0}{\sqrt{1 - \left(\frac{10^7 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)^2}} = \frac{t_0}{\sqrt{1 - \left(\frac{1}{30}\right)^2}}$$

$$24 = \frac{t_0}{0.99944} \Rightarrow t_0 = 24 \times 0.99944 \underbrace{\frac{1}{30}}_{= 0.0333}$$

\downarrow

$$t_0 = 23.9866 \text{ hours}$$

Hence loss in one day

$$= 24 - 23.9866 = 0.0134 \text{ h} \underbrace{\frac{1 - \left(\frac{1}{30}\right)^2}{1 - \left(\frac{1}{30}\right)^2}}_{= 0.99888} = 0.99944$$

$$= 0.804 \text{ m} = 48.24 \text{ sec}$$

Q: How fast must a spacecraft travel relative to the earth for each day on the space craft to correspond to 2 day on the earth?

$$v = ? , t_0 = 1 \text{ day} = 24 \text{ hours}$$

$$t = 2 \text{ day} = 2 \times 24 = 48 \text{ h.}$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 48 = \frac{24}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \Rightarrow \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{24}{48}$$

$$1 - \left(\frac{v}{c}\right)^2 = \frac{1}{4} \Rightarrow \left(\frac{v}{c}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\sqrt{3} = 1.732$$

$$v = \frac{\sqrt{3}}{2} c = 2.598 \times 10^8 \text{ m/s.}$$

$$v = 2.6 \times 10^8 \text{ m/s.}$$